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Problems in Time-Machine Construction due to Wormhole Evolution

Frank Antonsen[1] and Karsten Bormann[1]

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We take the dynamics of wormholes into account when discussing the construction of time machines. Criteria for time travel to be possible are derived.

1. INTRODUCTION

A wormhole can be brought to behave as a time machine either by accelerating the mouths relative to one another (at appropriate accelerations and then placing them at an appropriate distance) or by placing the mouths in gravitational potentials of differing values (and at an appropriate distance). The time shift is due to the fact that the clocks placed (at rest) at the wormhole mouths tick at different rates at least part of the wormhole's lifetime (Morris *et al.*, 1988). Recently, the authors have been investigating the influence of vacuum fluctuations of a scalar field (the Casimir effect) on the evolution of a hyperspatial tube model of topology $S^2 \times R \times R$, i.e., of wormhole topology (Antonsen and Bormann, n.d.). This model may be seen as an approximation to the central parts of a wormhole (Fig. 1); in particular, as the Einstein equations are local, it is likely that at least parts of a real-world wormhole would behave somewhat like the hyperspatial tube. Thus, when discussing the criteria for the wormhole to be a possible time machine, we take the behavior of the hyperspatial tube, presented in the next section, as our starting point.

2. WORMHOLE EVOLUTION SCENARIOS

When one takes into account the space-time dynamics as derived from vacuum fluctuations of quantum fields (the Casimir effect) and the Einstein

[1] Niels Bohr Institute, DK-2100 Copenhagen Ø, Denmark.

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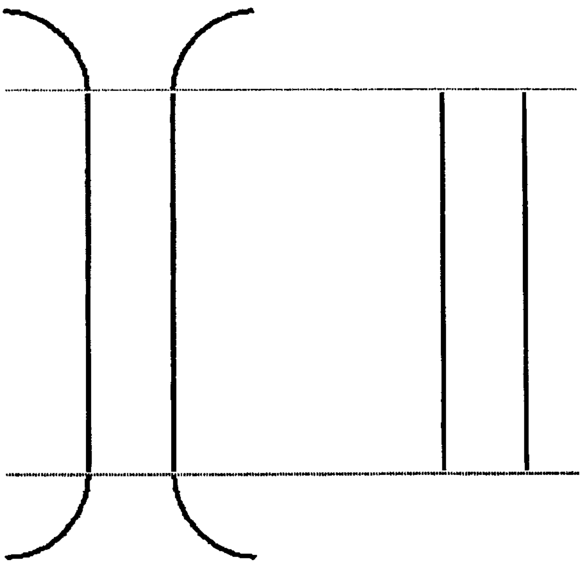

a b

Fig. 1.

equations, there are essentially four possibilities for the evolution of a hyperspatial tube,[2] which can be summarized by Fig. 2a–2c: (A) oscillatory behavior in the b direction and collapse in the z direction (Fig. 2a); (B) expansion in both z and b directions (Fig. 2b, early times); (C) expansion in the b direction with z becoming constant $\sim 10^{-21}$ m (Fig. 2b, later times); and finally (D) expansion in the z and collapse in the b direction. Now consider the consequences for real-world wormholes:

The assumption is that wormholes are born at the Planck scale because of fluctuations of the gravitational field. But then one can be pretty sure that a wormhole that collapses in both directions will also be eaten by the fluctuations. The wormholes oscillatory in the b direction (A) can be done away with by the same token as they collapse in z direction and also periodically become small (<1 Planck length) in the b direction. The discussion of the other scenarios is a little more complicated, as follows.

3. EXPANSION IN THE z DIRECTION (SCENARIO B)

As regards wormholes expanding in the z direction, the problem is in getting a negative time step, i.e., in getting backward in time. Let us discuss this with reference to early times in Fig. 2b where the expansion is explosive:

[2]For the choice of parameters investigated in Antonsen and Bormann (n.d.).

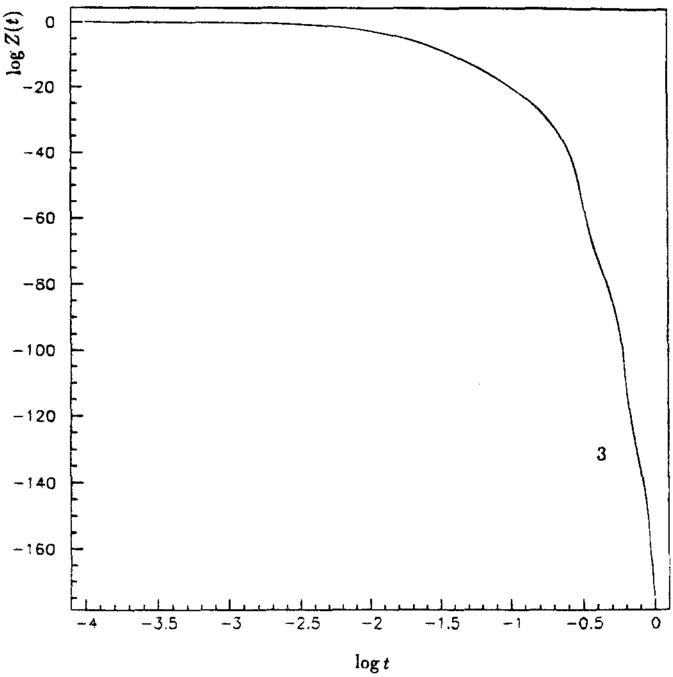

Fig. 2a.

As sketched in Fig. 3a, if the wormhole mouths are placed a distance d apart and the wormhole itself is of length l, then, in order to serve as a time machine, the time step when going through the wormhole has to be at least $(l + d)/c$, where c is the velocity of light ($c = 1$ in Fig. 2). It is normally assumed that l is small; however, this may not always be a sound assumption.

In the following we will take the evolution of the wormhole into account and try to determine the maximal time step one can go backward in time when going through an expanding (in the z direction) wormhole.

Note that when the time traveller is standing at the wormhole entrance, she should be able to traverse the wormhole at a velocity less than that of light and still get to the other end, i.e., she should see no part of the wormhole receding at $v_w > c$ (cf. Fig. 3b). Let us recast this formally:

Let us mark positions in the z direction in the wormhole by their z values at $t = 0$; denote them $Z_z(t = 0)$ at $t = 0$ and $Z_z(t)$ at $t > 0$. At $t = 0$ the time traveller is at the wormhole entrance, $Z_0(0)$. Now, what we have to do is to determine $\tilde{z} = Z_{\tilde{z}}(0)$ so that at this distance the wormhole is receding from the time traveller at the speed of light. This gives us

$$\frac{\Delta Z}{\Delta t} = \frac{Z_{\tilde{z}}(t) - Z_0(t)}{t} = \frac{Z_{\tilde{z}}(t)}{t} = c \qquad (1)$$

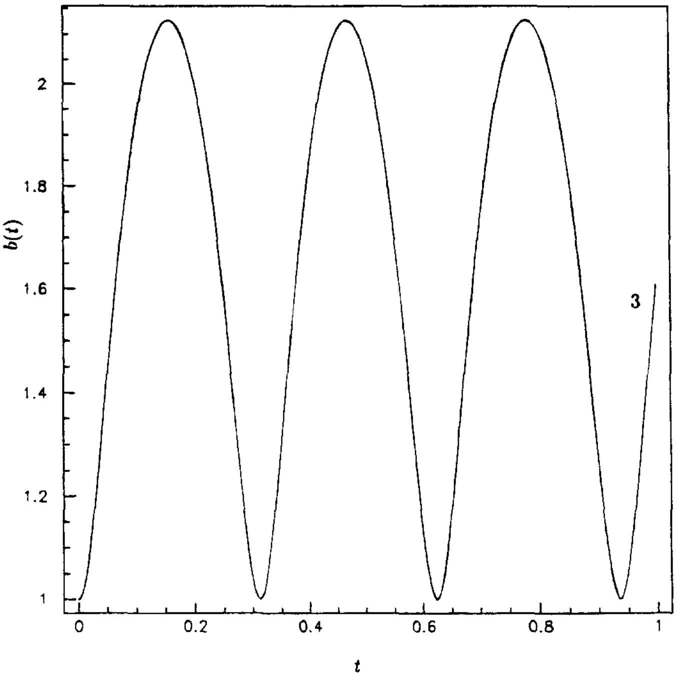

Fig. 2b.

so the value of $\tilde{z}$ which defines the horizon is

$$Z_{\tilde{z}}(t) = ct \tag{2}$$

Now the scale of the wormhole in the z direction is as in Fig. 2b. Thus we want to express the horizon in terms of the scale factor, as follows: Due to the (translation) symmetry (along the z axis) of the wormhole we have

$$Z_{\tilde{z}}(t) = \rho(t)Z_{\tilde{z}}(0) = \rho(t)\tilde{z} \tag{3}$$

where $\rho(t)$ is the (z-independent) scale factor. Thus the horizon is given by

$$\tilde{z} = \frac{ct}{\rho_z(t)} \tag{4}$$

In Fig. 2 time and length are measured in Planck units (10^{-43} sec and 10^{-33} m, respectively) and one therefore see that, at least at early times, the expansion is so explosive that the wormhole exit will be outside the time traveller's horizon. However, it need not be outside her horizon forever. Actually, for large times the wormhole often expands according to a power law or its length becomes constant. Whether or not the other wormhole mouth stays outside of the horizon depends on the size of the wormhole when

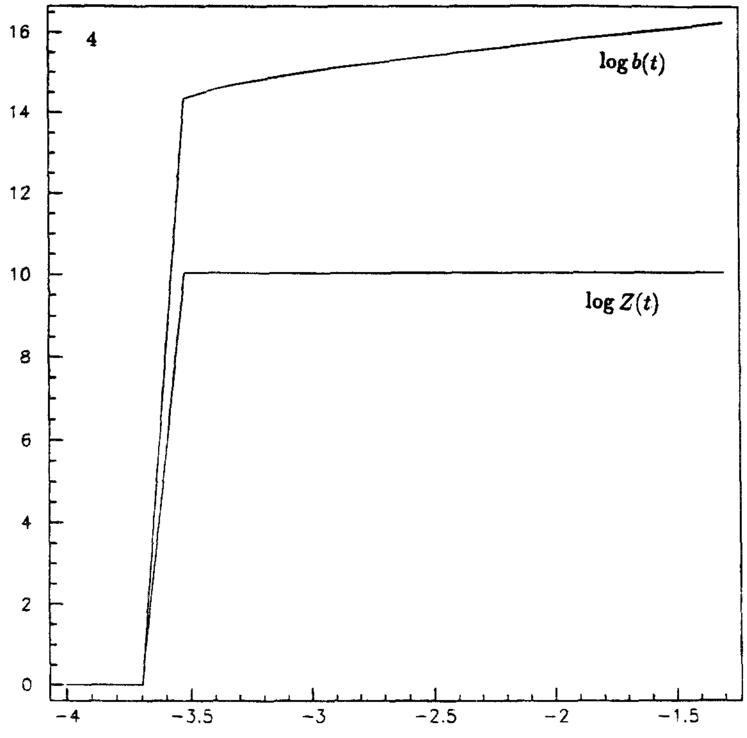

Fig. 2c.

"inflation" stops and on the power of the scale factor expansion: Call the length of the wormhole at the end of inflation L; then the other wormhole mouth remains inaccessible provided that

$$\partial_t \rho \cdot L \geq c \tag{5}$$

which, incidentally, does not hold for Fig. 2b.

4. EXPANSION IN THE *B* DIRECTION (SCENARIOS B, C, D)

It thus seems that the only hope is for the D and (especially) the C scenarios to provide us with useful wormholes, and because they are rather alike, we will discuss the usefulness of both as follows:

First note that if again one think of the hyperspatial tube as an approximation to the central parts of the wormhole only, then there are two possibilities. Either the mouths expand along with the central parts or they do not. The two cases are sketched in Figs. 4a and 4b. Because of the extreme expansion

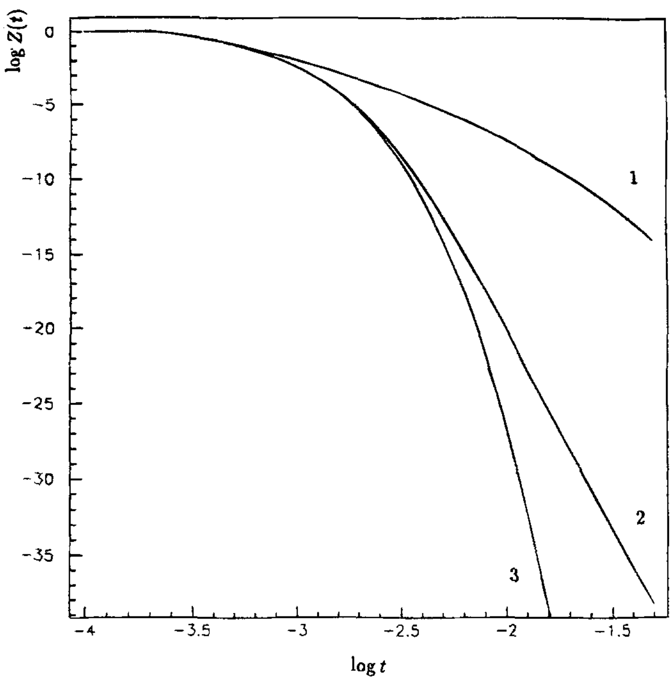

Fig. 2d.

of the "radius" of the central parts of the wormhole the discussion of scenario B applies to the case of Fig. 4b.

As regards the scenario of Fig. 4a, we will distinguish between creating the time machine by placing one of the wormhole mouths in the neighborhood of a black hole and by accelerating the wormhole mouths relative to one another: When one creates a time machine by accelerating the mouths relative to one another, then one moves them away from one another, at speeds close to c, and then has to put them back in place. But if the mouths are at the same time expanding at (geometrical) speed $\geq c$, then this cannot be done.

In the neighborhood of a black hole the situation is as sketched in Fig. 5. The argument in a nutshell is that because of the expansion in the b direction, we only have a limited time before the black hole would coalesce with wormhole mouth 1, giving only a finite time to create the time step: So we want to be close to the black hole to create a time step, but we do not want to be too close in order not to have the black hole *inside* the wormhole. Formally the argument is as follows: We must have that $r - b > 2m$, where $b = \rho(t)b_0$. Furthermore, as $dt = [1 - (2m/r)\, d\tau]^{1/2}$ and because it takes at least the time $(d + l)/c$ to get from wormhole mouth 1 through

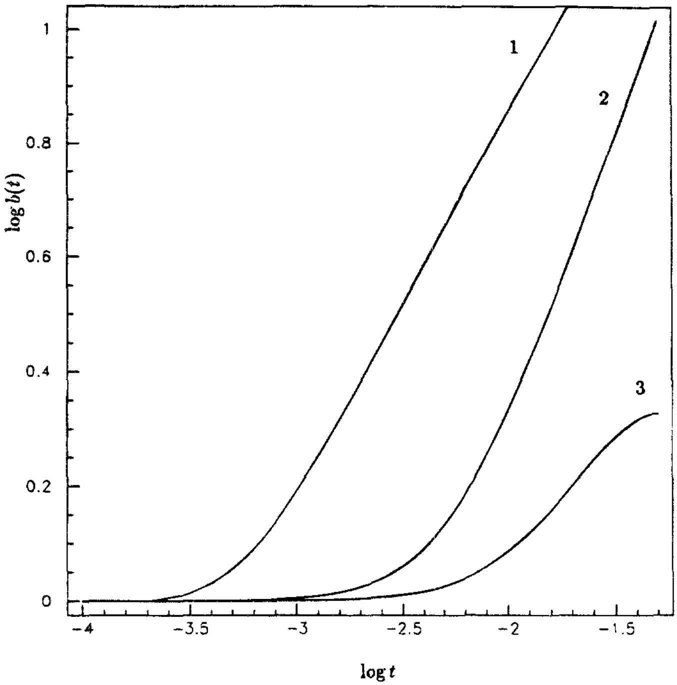

Fig. 2e.

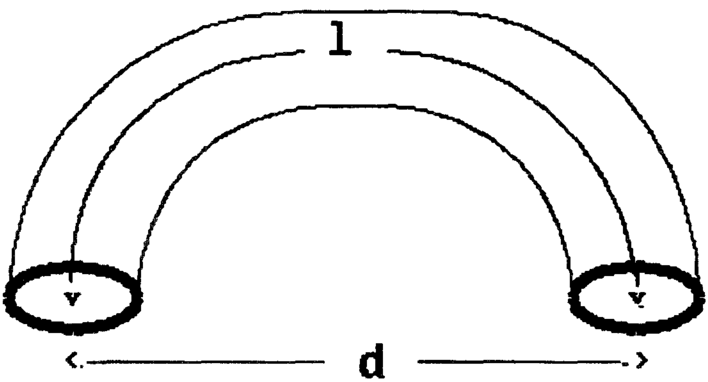

Fig. 3a.

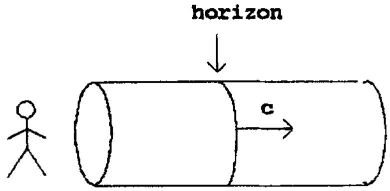

Fig. 3b.

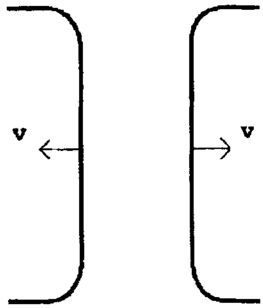

Fig. 4a.

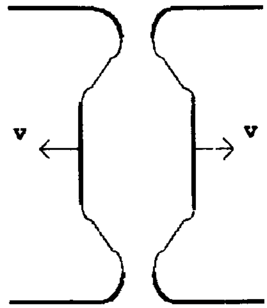

Fig. 4b.

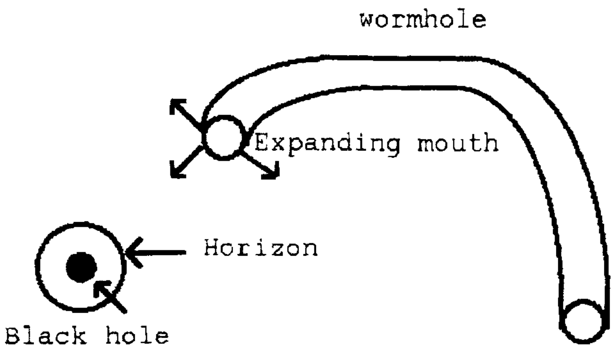

Fig. 5.

the wormhole and back through ordinary spacetime, the time step of the time machine can be no greater than

$$\left[\left(1 - \frac{2m}{r + d}\right)^{1/2} - \left(1 - \frac{2m}{r}\right)^{1/2}\right]\tau - \frac{d + l}{c} \tag{6}$$

(where $c = 1$ in Fig. 2), calling the wormhole's time of birth $t = \tau = 0$. This quantity has to be positive in order for the wormhole to function as a time machine. Furthermore, as the wormhole eats the black hole at time $\tau \sim \rho^{-1}((r - 2m)/b_0)$ and as $d < c\tau$, this time step is small, if not negative, because gravity is weak (but of course m dependent) and the expansion of the wormhole mouth rather large.

Anyway, to exist, the majority of type 4a wormholes should be huge today, but they are conspicuously absent from observations.

5. CONCLUSION

We have noted the relevance of wormhole dynamics (as derived from the Casimir effect and the Einstein equations) to time machine construction and then took the evolution of the model space-time of Fig. 1 as a starting point for deriving conditions for a dynamically evolving wormhole to function as a time machine. If the results summarized in Fig. 2 are indicative for the behavior of real-world wormholes (which we think they are, because the topology of model is the same as that of a wormhole and because the Einstein equations are local), then these conditions may be difficult to meet, even if the collapse in the z direction that normally occurs is nice from the point of view of the wormhole constructor. If, however, no particles exist heavier than $\sim 10^3$ GeV, then this would correspond to choice of parameters in scenario C. This is by far the most promising in that they look somewhat like standard wormholes. The expansion, however, poses great difficulties in making the wormhole behave like a time machine.

REFERENCES

Antonsen, F., and Bormann, K. (1995). The evolution of a hyperspatial tube due to the Casimir effect, *Physical Review D*, **51**, 568–578.

Morris, M. S., Thorne, K. S., and Yurtsever, U. (1988). *Physical Review Letters D*, **61**, 1446.